CONTACT THERMAL CONDUCTIVITY OF COMPRESSIBLE ROUGH SURFACES

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An analytical relationship connecting the thermal conductivity of elastic, elastic–plastic, and plastic contacts of rough surfaces with the microgeometric and strength properties, and also with the external mechanical load, is obtained.

An analytical relationship between the contact conductivity and the external load applied to rough surfaces was obtained in [1]. This relationship was based on the familiar premise that the contact of two rough surfaces can be regarded as the contact of a smooth surface with a hypothetical rough surface for which the variance of the projection height distribution is known. In the case of real compression the roughness projections on both surfaces undergo deformation. We will show below how we can take into account these joint deformations in finding the relationship $\alpha(P)$.

We consider the contact of rough surfaces on which the projections have tip radius R_i and heights z_i relative to a reference plane M_i, which conform, according to profilometric measurements, to a Gaussian distribution $f(z_i)dz_i$ with variance σ_i^2 and N_i roughness projections per unit area of reference plane.

On application of the load and initial distance between M_i, equal approximately to $d^* \approx z_1^* + z_2^*(z_i^* \approx 4\sigma_i)$, is reduced by a value $\delta = \delta_1^* + \delta_2^*$ and becomes equal to $d = d_1 + d_2$. It follows from [2] that

$$\delta_i = \frac{3}{4}\,\frac{1-\mu_i^2}{E_i}\,F^{2/3}\left(\frac{1}{D}\,\frac{R_1+R_2}{R_1R_2}\right)^{1/3},$$

$$D = \frac{3}{4}\left(\frac{1-\mu_1^2}{E_1} + \frac{1-\mu_2^2}{E_2}\right). \quad (1)$$

It is known that the thermal conductivity of a single contact spot is

$$\alpha_n = \left(\frac{1}{\pi\lambda a}\ \mathrm{arctg}\,\frac{r-a}{a}\right)^{-1}, \qquad \lambda = \frac{2\lambda_1\lambda_2}{\lambda_1+\lambda_2}. \quad (2)$$

Since the contact spots can be regarded as uniformly distributed, then $r = (1/2)\sqrt{1/n}$. The number n depends on the clearance between the surfaces if the latter are conjugate-cylindrical, or on the external mechanical load applied to the surfaces if these surfaces are flat. This number is found from the following considerations. If the rough surface with i = 1 is compressed by a smooth surface, the number of contact spots is

$$n = \frac{N_1}{\sigma_1\sqrt{2\pi}}\int\limits_{d_1}^{z_1^*}\exp\left(-\frac{z_1^2}{2\sigma_1^2}\right)dz_1, \qquad d_i = z_i^* - \delta_i^*. \quad (3)$$

However, since the surface with i = 2 is also rough, to obtain n we have to multiply expression (3) by the probability that the projections of the surface with i = 1 meet the projections of the surface i = 2. This probability is equal to the relative actual area of contact of the surface with i = 2 when its highest roughness projections undergo a deformation δ_2^* due to a smooth surface, whence

$$n = n_1\int\limits_{d_2}^{z_2^*}\pi a_2^2 f(z_2)\,dz_2. \quad (4)$$

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For an elastic contact of a spherical projection with a smooth surface $a_2 = \sqrt{R_2(z_2-d_2)}$ and, hence, after calculations

$$n=\frac{1}{2}\sqrt{\frac{\pi}{2}}R_2N_1N_2A_2\,\mathrm{erfc}\left(\frac{d_1}{\sqrt{2}\,\sigma_1}\right), \tag{5}$$
$$A_i\equiv\sigma_i\exp\left(-\frac{d_i^2}{2\sigma_i^2}\right)-\sqrt{\frac{\pi}{2}}d_i\,\mathrm{erfc}\left(\frac{d_i}{\sqrt{2}\,\sigma_i}\right),$$

where $\mathrm{erfc}\,x = 1-\Phi(x)$, and $\Phi(x)$ is the probability integral.

The thermal conductivity of unit area of a nominal contact surface, which is the integral sum of the conductivities via individual contact spots, is also found by taking into account the above considerations, i.e.,

$$\alpha=\int_{d_1}^{z_1^*}\int_{d_2}^{z_2^*}\alpha_n\pi R_2(z_2-d_2)f(z_1)f(z_2)\,dz_1dz_2. \tag{6}$$

It follows from [2] that the radius a in expression (2) for an elastic contact is

$$a=\sqrt{\frac{R_1R_2}{R_1+R_2}\left[(z_1-d_1)+(z_2-d_2)\right]}, \tag{7}$$

and, hence, using the inequality $r \gg a$ we find from (6), after calculations, that

$$\alpha=\lambda N_1N_2R_2\sqrt{\frac{R_1R_2}{R_1+R_2}}\left\{\sqrt{z_2^*-d_2}\left[\frac{4}{9}\,\frac{1-\mu_2^2}{1-\mu_1^2}\,\frac{E_1}{E_2}\right.\right.$$
$$\left.\left.+\frac{1}{\pi r}\sqrt{\frac{R_1R_2}{R_1+R_2}(z_2^*-d_2)}\right]\sqrt{\frac{\pi}{2}}A_2\,\mathrm{erfc}\left(\frac{d_1}{\sqrt{2}\,\sigma_1}\right)+\frac{2}{\pi r}\sqrt{\frac{R_1R_2}{R_1+R_2}}A_1A_2\right\}. \tag{8}$$

Taking the statistical properties of the contact into account we can express the mechanical load in terms of the deformations of the microprojections. Using relationship (1) we find that for an elastic contact

$$P=\frac{1}{D}\sqrt{\frac{R_1R_2}{R_1+R_2}}\int_{d_2}^{z_2^*}\pi R_2(z_2-d_2)f(z_2)\,dz_2\int_{d_1}^{z_1^*}(z_1-d_1+z_2-d_2)^{3/2}f(z_1)\,dz_1,$$

whence it follows that

$$P=\frac{2}{9}\gamma(1+\gamma)\frac{R_2}{D}N_1N_2A_1A_2\sqrt{\frac{R_1R_2}{R_1+R_2}(z_2^*-d_2)},$$
$$\gamma\equiv\frac{1-\mu_2^2}{1-\mu_1^2}\,\frac{E_1}{E_2}. \tag{9}$$

An additional relationship between d_1 and d_2, obtained from the condition of mutuality of the stresses at contact [2], has the form

$$\left(\frac{E_1}{1-\mu_1^2}\right)^{3/2}\sqrt{z_1^*-d_1}\,\sigma_1A_1=\left(\frac{E_2}{1-\mu_2^2}\right)^{3/2}\sqrt{z_2^*-d_2}\,\sigma_2A_2. \tag{10}$$

The contact thermal conductivity, with due allowance for the radiative heat transfer and the intervening medium, is given by $\bar{\alpha} = \alpha + (1-\varepsilon)(\alpha_r + \alpha_i)$. The relative actual area of contact ε in this expression is the integral sum of the areas of the individual contact spots. For an elastic contact

$$\varepsilon_{el}=\frac{\pi}{2}(1+\gamma)\frac{R_1R_2^2}{R_1+R_2}N_1N_2A_1A_2. \tag{11}$$

For a plastic contact α is $\sqrt{2}$ times greater than the value given by expression (8), and ε_{pl} is twice as great as ε_{el}. According to the well-known Bowden and Tabor relationship the load applied to the plastic contact is

$$P_{pl} = \pi (1+\gamma) \frac{R_1 R_2^2}{R_1 + R_2} N_1 N_2 A_1 A_2 H. \qquad (12)$$

In all the calculations given above we took into account that the properties of the Gaussian distribution allow the upper limit of integration z_i^* to be replaced by infinity.

Thus, we have obtained analytical relationships $\bar{\alpha}(d_1, d_2)$ and $P(d_1, d_2)$ which, as distinct from [1], contain the strength and microgeometric characteristics of the two contacting surfaces. It is obvious that this is equivalent to finding the relationship $\bar{\alpha}(P)$. The dependences of α, P, and ε on d_1 and d_2 for elastic –plastic contact are given by relationships of the type $\varepsilon = (1-\beta)\,\varepsilon_{el} + \beta\varepsilon_{pl}$. The plasticity coefficient β is given in [1].

NOTATION

α	is the contact thermal conductivity;
P	is the external mechanical load;
R, z, and z*	are the radius of tip, height, and maximum height of roughness projections;
N and n	are the number of roughness projections and number of contact spots per unit area of reference plane;
i = 1, 2	is the subscript denoting contacting surfaces;
σ	is the standard deviation of projection heights;
E and μ	are the modulus of elasticity and Poisson's ratio for material of projection;
F and δ	are the load on projection and its deformation;
δ^*	is the deformation of highest roughness projections (maximum deformation);
a and r	are the radii of contact spot and region of compression;
λ	is the effective thermal conductivity of contact zone;
a_i	is the radius of contact spot in case of compression of roughness projection of i-th surface by smooth surface;
ε	is the relative actual contact area;
α_i	is the heat transfer coefficient of intervening medium;
α_r	is the radiative heat transfer coefficient;
H	is the hardness of less hard surface.

LITERATURE CITED

1. V. S. Novikov, Inzh.-Fiz. Zh., 19, No. 1, 62 (1970).
2. L. D. Landau and E. M. Lifshits, Theory of Elasticity, Nauka, Moscow (1965).